

Hong Kong Mathematics Olympiad (2001 – 2002)

Heat Event (Individual)

香港數學競賽 (2001 – 2002)

初賽項目(個人)

除非特別聲明，答案須用數字表達，並化至最簡。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 袋中有數字卡 9 張，其數字分別為 1 至 9。若隨機一次抽出 3 張，求被抽出的卡的數字全是奇數的概率。(答案以分數表達，並化至最簡。)

There are 9 cards, numbered from 1 to 9, in a bag. If 3 cards are drawn together at random, find the probability that all are odd. (Express your answer in the simplest fraction.)

2. 已知 $a^3 = 150b$ ，且 a 和 b 都是正整數。求 b 的最小值。

Given $a^3 = 150b$, and a, b are positive integers, find the least value of b .

3. 已知 $\cos 15^\circ = \frac{\sqrt{a} + \sqrt{b}}{4}$ ，且 a, b 是自然數。若 $a + b = y$ ，求 y 的值。

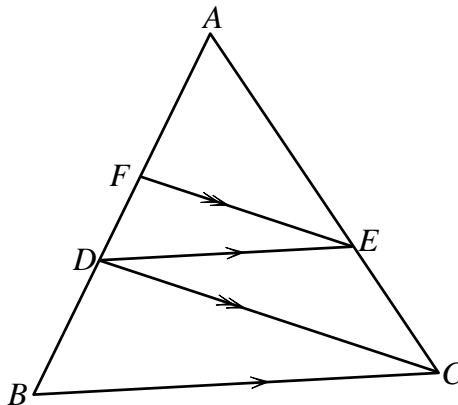
Suppose $\cos 15^\circ = \frac{\sqrt{a} + \sqrt{b}}{4}$, and a, b are natural numbers. If $a + b = y$, find the value of y .

4. 把數字 2, 3, 4, 5 組成沒有重複數字的四位數，求這些四位數的和。

Each of the digits 2, 3, 4, 5 can be used once and only once in writing a four-digit number. Find the sum of all such numbers.

5. 在 $\triangle ABC$, $DE \parallel BC$, $FE \parallel DC$, $AF = 2$, $FD = 3$ 和 $DB = X$ 。求 X 的值。

In $\triangle ABC$, $DE \parallel BC$, $FE \parallel DC$, $AF = 2$, $FD = 3$ and $DB = X$. Find the value of X .



6. 若一圓內接四邊形的四邊長度為 9 , 10 , 10 和 21 , 求該圓內接四邊形的面積。

If the lengths of the sides of a cyclic quadrilateral are 9, 10, 10 and 21 respectively, find the area of the cyclic quadrilateral.

7. 若 $\frac{(a-b)(c-d)}{(b-c)(d-a)} = 3$, 求 $\frac{(a-c)(b-d)}{(a-b)(c-d)}$ 的值。

If $\frac{(a-b)(c-d)}{(b-c)(d-a)} = 3$, find the value of $\frac{(a-c)(b-d)}{(a-b)(c-d)}$.

8. 若 $x^3 + kx^2 + 3$ 除以 $x+3$, 其餘數較被 $x+1$ 除所得的餘數少 2 。求 k 的值。

When the expression $x^3 + kx^2 + 3$ is divided by $x+3$, the remainder is 2 less than when divided by $(x+1)$. Find the value of k .

9. 已知圓形上的某扇形的周界為 18 。當圓的半徑為 r 時 , 該扇形的面積達至最大值 , 求 r 的值。

Given that the perimeter of a sector of a circle is 18. When the radius is r , the area of the sector attains the maximum value, find the value of r .

10. 已知 $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$ ，求 $f(5)$ 的值。

Given $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$, find the value of $f(5)$.